Scholarly Research Journal for Interdisciplinary Studies,
Online ISSN 2278-8808, SJIF 2021 = 7.380, www.srjis.com
PEER REVIEWED & REFEREED JOURNAL, JULY-AUGUST, 2021, VOL- 9/66
10.21922/srjis.v9i66.6849

ON MULTIPLICATIVE K BANHATTI INDICES OF LINE GRAPHS

B. Manjunath

Department of Mathematics, Governement First Grade College, Mulbagal, Kolar-563 101

Paper Received On: 21 JULY 2021

Peer Reviewed On: 31 JULY 2021

Published On: 1 SEPT 2021

Abstract

Let $G = (V,E)$ be a connected graph. The multiplicative K Banhatti indices of G are defined as $B\Pi_*(G) = {}^{Q}{}_{ue}[d_G(u) * d_G(e)]$, where $*$ is usual addition or multiplication and ue means that the vertex u and edge e are incident in G. In this paper, we compute the multiplicative K Banhatti indices of line graphs..

Mathematics Subject Classification: *05C05, 05C07, 05C35.*

Keywords: *Multiplicative K Banhatti indices; Line graph.*

1 Introduction

By a graph, we mean a finite, undirected without loops and multiple edges. Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. Let $d_G(e)$ denotes the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$ with $e = uv$. For definitions and notions, the reader may refer to [7].

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. In Chemical Science, the physico- chemical properties of chemical compounds are often modeled by means of molecular graph based structure descriptors, which are also referred to as topological indices, see [3].

The K Banhatti indices of G are defined as $B_*(G) = {}^{P}{}_{ue}[d_G(u) * d_G(e)]$, where $*$ is usual addition or multiplication and ue means that the vertex u and edge e are incident in G. If $*$ is addition, then first K Banhatti index $B_+(G) = B_1(G) = {}^{P}{}_{ue}[d_G(u) + d_G(e)]$, and if $*$ is multiplication, then the second K Banhatti index $B_\times(G) = B_2(G) = {}^{P}{}_{ue}[d_G(u) \times d_G(e)]$. The K Banhatti indices were introduced by Kulli in [8].

Todeschini et al. [12] proposed that multiplicative variants of molecular structure descriptors be considered. When this idea is applied to Zagreb indices, for example, in [2]. The multiplicative version of first and second K Banhatti indices were introduced by Kulli in [10] and [11] as follows.

The multiplicative K Banhatti indices of G are defined as $B\Pi_*(G) = \prod_{ue}[d_G(u) * d_G(e)]$, where $*$ is usual addition or multiplication and ue means that the vertex u and edge e are incident in G. If $*$ is addition, then first multiplicative K Banhatti index $B\Pi_+(G) = B\Pi_1(G) = \prod_{ue}[d_G(u) + d_G(e)]$, and if $*$ is multiplication, then the second multiplicative K Banhatti index $B\Pi_\times(G) = B\Pi_2(G) = \prod_{ue}[d_G(u)d_G(e)]$. Recently many other indices were studied, for example, in [4] and [9].

The Line graph $L(G)$ is the graph with vertex set $V(L) = E(G)$ and whose vertices correspond to the edges of G with two vertices being adjacent if and only if the corresponding edges in G have a vertex in common two. For more

details, we refer to [5].

2 Results

Theorem 2.1 *Let G be a r- regular graph with $n \geq 2$ vertices. Then*

(i) $B\Pi_1(L(G)) = [2(3r-4)]^{nr(r-1)}$,

(ii) $B\Pi_1(L(G)) = [4(r-1)(2r-3)]^{nr(r-1)}$.

Proof.

Let G be a r- regular graph with $n \geq 2$ vertices. By algebraic method, we have $|V(L(G))| = \frac{nr}{2}$ and $|E(L(G))| = \frac{nr}{2}(r-1)$. Since line graph of a r - regular graph is $(2r-2)$ - regular and $B\Pi_*(L(G)) = \prod_{ue}[d_{L(G)}(u) * d_{L(G)}(e)]$. Hence, we have the following cases:

Case 1. $B\Pi_+(L(G)) = B\Pi_1(L(G)) = \prod_{ue}[d_{L(G)}(u) + d_{L(G)}(e)]$

$$= \left[(2r-2+4r-6)^2\right]^{\frac{1}{2}nr(r-1)}$$
$$= [2(3r-4)]^{nr(r-1)}.$$

Case 2. $B\Pi_\times(L(G)) = B\Pi_2(L(G)) = \prod_{ue}[d_{L(G)}(u) \times d_{L(G)}(e)]$

$$= \left[[(2r-2)(4r-6)]^2\right]^{\frac{1}{2}nr(r-1)}$$
$$= [4(r-1)(2r-3)]^{nr(r-1)}.$$

Thus the result follows.

By above Theorem, we have the following result without proof.

Theorem 2.2 *Let G be a r- regular graph with $n \geq 2$ vertices. Then*

$$B\Pi_2(G)(L(G)) = \left[\frac{2(r-1)(2r-3)}{(3r-4)}\right]^{nr(r-1)} B\Pi_1(G)(L(G)).$$

Corollary 2.3 *Let C_n be a cycle with $n \geq 3$ vertices. Then*

$B\Pi_1(L(C_n)) = B\Pi_2(L(C_n)) = 4^{2n}$.

Corollary 2.4 *Let K_n be a complete graph with $n \geq 3$ vertices. Then*

(i) $B\Pi 1(L(Kn)) = (6n - 14)n(n-1)(n-2)$,

(ii) $B\Pi_1(L(K_n)) = [4(n-2)(2n-5)]^{n(n-1)(n-2)}$.

Theorem 2.5 *Let P_n be a path with $n \geq 4$ vertices. Then*

(i) $B\Pi_1(L(P_n)) = 9 \times 2^{4n-14}$, *(ii)*

$B\Pi_1(L(P_n)) = 2^{4n-14}$.

Proof.

Let P_n be a path with $n \geq 4$ vertices. Since $L(P_n) \cong P_{n-1}$. By algebraic method, we have $|V(L(P_n))| = n-1$ and $|E(L(P_n))| = n-2$. We have two partitions of the vertex set $V(L(P_n))$ as follows: $V_1 = \{v \in V(L(P_n)) : d_{L(Pn)}(v) = 1\}; |V_1| = 2$, and

$V_2 = \{v \in V(L(P_n)) : d_{L(Pn)}(v) = 2\}; |V_2| = n-3$.

Also we have two partitions of the edge set $E(L(P_n))$ as follows:

$E_1 = \{uv \in E(L(P_n)) : d_{L(Pn)}(u) = 1, d_{L(Pn)}(v) = 2\}; |E_1| = 2$, and

$E_2 = \{uv \in E(L(P_n)) : d_{L(Pn)}(u) = d_{L(Pn)}(v) = 2\}; |E_2| = n-4$.

Then $B\Pi*(L(Pn)) = Que[dL(Pn)(u) * dL(Pn)(e)]$

$$= \prod_{uv\in E_1} [dL(Pn)(u) * dL(Pn)(e)] + \prod_{uv\in E_2} [dL(Pn)(u) * dL(Pn)(e)]$$

We have the following two cases are arise:

Case 1. $B\Pi_+(L(P_n)) = B\Pi_1(L(P_n))$

$$= \prod_{uv\in E_1} [(1+1) \times (2+1)] \times \prod_{uv\in E_2} [(2+2) \times (2+2)]$$

$$= (2 \times 3)^2 \times (4 \times 4)^{n-4} = 9 \times 24n-14.$$

Case 2. $B\Pi_\times(L(P_n)) = B\Pi_2(L(P_n))$

$$= \prod_{uv\in E_1} [(1 \times 1) \times (2 \times 1)] \times \prod_{uv\in E_2} [(2 \times 2) \times (2 \times 2)]$$

$= (1 \times 2)^2 \times (4 \times 4)^{n-4} =$

$24n-14.$

Thus the result follows.

By above Theorem, we have the following result without proof.

Theorem 2.6 *Let P_n be a path with $n \geq 4$ vertices. Then*

$B\Pi_1(L(P_n)) = 9\, B\Pi_2(L(P_n)).$

Corollary 2.7 *Let C_n be a cycle and P_n be a path with $n \geq 4$ vertices.*

Then

(i) $B\Pi_1(L(P_n)) = 9 \times 2^{-14} B\Pi_1(L(C_n)),$ *(ii)*

$B\Pi_2(L(P_n)) = 2^{-14}\, B\Pi_2(L(C_n)).$

Theorem 2.8 *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$ ver-*

tices. Then

(i) $B\Pi 1(L(Kr,s)) = [3r + 3s - 8]rs(r+s-2),$

(ii) $B\Pi_2(L(K_{r,s})) = [(r + s - 2)(2r + 2s - 6)]^{rs(r+s-2)}.$

Proof.

Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$ vertices. By algebraic method, we have $|V(L(K_{r,s}))| = rs$, and $|E(L(K_{r,s}))|$ =. Since line graph of complete bipartite graph $K_{r,s}$ is a $(r+s-2)$-regular graph and $B\Pi_*(L(K_{r,s})) =$

$Que[dL(Kr,s)(u) * dL(Kr,s)(e)]$. We have

(i) $B\Pi_+(L(K_{r,s})) = B\Pi_1(S(K_{r,s}))$

$$= \left[[(r+s-2)+(r+s-2+r+s-2-2)]^2\right]^{\frac{1}{2}rs(r+s-2)}$$

$$= [3r+3s-8]^{rs(r+s-2)}.$$

(ii) $B\Pi_\times(L(K_{r,s})) = B\Pi_2(L(K_{r,s}))$

$$= \left[[(r+s-2)\times(r+s-2+r+s-2-2)]^2\right]^{\frac{1}{2}rs(r+s-2)}$$

$$= [(r+s-2)(2r+2s-6)]^{rs(r+s-2)}.$$

The following results are immediate from above theorem.

Corollary 2.9 *Let $K_{1,s}$ be a star graph with $s \geq 1$ vertices. Then*

(i) $B\Pi_*(L(K_{1,s})) = B\Pi_*(K_s),$

(ii) $B\Pi_1(L(K_{1,s})) = (3s - 5)^{s(s-1)},$

(iii) $B\Pi_2(L(K_{1,s})) = [2(s - 1)(s - 2)]^{s(s-1)},$

Corollary 2.10 *Let $K_{r,r}$ be a regular complete bipartite graph with $r \geq 2$ vertices. Then*

$$B\Pi_2(L(K_{r,r})) = \left[\frac{2(r-1)(2r-3)}{3r-4}\right]^{2r^2(r-1)} B\Pi_1(L(K_{r,r})).$$

References

M. Behzad and G. Chartrand, An Introduction to Total graphs, Coloring, Line graphs., Proc. Symp. Rome. (1966), 31-33.

I. Gutman, Multiplicative Zagreb indices of trees, Bull. Internat. Math. Virt. Inst. 1 (2011), 13-19.

I. Gutman and N. Trinajstic, Graph Theory and molecular orbitals, Total π-electron energy of alternant hydrocarbons, Chem. Phys. Lett. 17 (1972), 535-538.

I. Gutman, V. R. Kulli, B. Chaluvaraju and H. S. Boregowda, On Banhatti and Zagreb Indices. J. Int. Math. Virtual Inst. 7 (2017), 53-67.

R. L. Hemminger and L. W. Beineke, Line graphs and line digraphs, in L. W. Beineke and R. J. Wilson, Selected Topics in Graph Theory, Academic Press Inc., (1978), 271-305.

S. R. Jog, S. P. Hande, I. Gutman and S. Burcu Bozkurt, Derived graphs of some graphs, Kragujevac Journal of Mathematics, 36 (2), (2012), 309-
314.

V. R. Kulli, College Graph Theory, Vishwa International Publications, Gulbarga, India (2012).

V. R. Kulli, On K Banhatti indices of graphs, Journal of Computer and Mathematical Sciences, 7(4), (2016), 213-218.

V. R. Kulli, On K indices of graphs, International Journal of Fuzzy Mathematical Archive, 10(2), (2016), 105-109.

V. R. Kulli, First Multiplicative K Banhatti index and coindex of graphs,
Annals of Pure and Applied Mathematics, 11 (2), (2016), 79-82.

V. R. Kulli, The Second multiplicative K Banhatti index and coindex of graphs, Journal of Computer and Mathematical Sciences, 7 (5), (2016), 254-258.

R. Todeschini, V. Consonni, New local vertex invariants and molecular descriptors based on functions of the vertex degrees, MATCH Commun.
Math. Comput. Chem. 64 (2010), 359-372.